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Volatility Forecasts and Value-at-Risk estimation using TGARCH model

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Abstract

Value-at-Risk (VaR) has emerged in recent years as a standard tool to measure and control the risk, mainly the market risk, of financial portfolios. It measures the worst loss to be expected of a portfolio over a given time horizon at a given level of confidence. The calculation of Value-at-Risk commonly, involves estimation of the volatility return price and quantile of standardized returns.

In this paper, two parametric techniques were used to estimate the volatility of the returns (market prices) of a Portuguese Financial Institution portfolio. Although to achieve the quantiles of standardized returns, both parametric technique and one nonparametric technique were considered. The quality of the measuring result was analysed through the backtesting technique for the forecasting multiperiod.

In this study it is revealed that AR(1)-TGARCH methodology produces the most accurate VaR forecast, for one day holding period. The volatility forecasts for the two other holding periods, considering the three methodologies, revealed to be biased.

Key words: Market Risk, Value-at-Risk, Volatility, Forecasting, TGARCH, Backtesting

JEL Classification: C14, C22, C52, G32

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1. Introduction

During the 90's, the financial world watched the fail of many large institutions, see Jorion (1997), due to the exposures to specific movements in the financial market. The instability in emerging markets, starting in Mexico in 1995, continuing in Asia in 1997, and spreading to Russia and Latin America in 1998, plotted the interest in Risk Management best practice.

These financial disasters brought clearly, to the financial world, the need to control financial risks. The regulatory authorities imposed the Risk-Based-Capital adequacy requirements on financial institutions (see Dimson and Marsh, 1995; Wagster, 1996). Consequently, good measures of risk have come into focus, and Risk Management became of supreme importance in the finance industry, especially for Institutional Investors such as Pension Funds, Insurance Companies and as well Asset Management Firms that manage funds on their one.

The need to determine the amount of risk, achieve the sense of the possibility of losses and measuring the risk accurately, turned out to be a critical matter for Financial Institutions.

The possible extent of a loss caused by an adverse market movement over the next day or next few days/months given the current volatility background, generally associated with the market risk of a given portfolio, became, especially for Risk Managers, the main challenge and an important concern.

For Risk Managers the main question turns out to be: how can we quantify risk; how much capital do we need to cover the risks under our business?

Theoretically, there are several possibilities: standard deviation, quantiles, interquantiles range or shortfall measures. Value-at-Risk (VaR), a quantile measure, has been the preferred tool in financial industry. Following the 1995 Basle Committee agreement, the Value-at-Risk (VaR) has turned into the standard risk measure adopted to define the market risk exposure of a financial position.

Nowadays, for Insurance Companies, Value-at-Risk (VaR) is a standard tool which quantify, with a certain confidence level, for a certain time period, the maximum anticipated loss in portfolio value due to adverse market movements. As providers of financial security the need to control the financial risk grows in order to provide protection and economic security to policyholders.

The actual European Capital Solvency Requirement, regulating the insurance activity, is not sufficiently sensitive to risk. A new risk based regulatory framework is under developing - Solvency II (see Swiss Re, Sigma n. º4, 2006). This new system gives special attention to the development of internal models which identify and capture the principal risk factors under the insurance activity (market, underwriting, operational and credit risk). Market Risk is, generally, the highest risk for a life Insurance Company.

Using a real Portuguese Life Insurance portfolio, this dissertation aims to measure and evaluate the quality of the volatility forecasting for Value-at-Risk (VaR) estimation using three different methodologies, two parametric techniques (RiskMetrics and TGARCH) and one non parametric technique (Historical Simulation).

Value-at-Risk was estimated for two different confidence levels, $\alpha = 5\%$ and $\alpha = 1\%$ for three forecasting holding time periods; one day, one week and two weeks.

The three methodologies revealed to be a good VaR estimator for one day forecasting period, excepting Risk Metrics with 1% of confidence level. There is evidence that for all methodologies the final results are biased for the forecasting holding periods, five and ten days.

This study is structured as follows: Section 2 categorizes parts of the existing literature under the methodology. The theoretical background and data analysis are described on section 3. On section 4 the estimation results are presented. Section 5 summarises the main findings.

2. Literature Review

The development of models for measuring forecasting volatility began with Engle (1982). Many findings resulting of Engle's original work had huge implications on actual risk management techniques.

One of those was the important contribution of the RiskMetrics by J.P.Morgan (1996) methodology: the introduction of the Value-at-Risk (VaR) concept. The new concept, as mention by Christoffersen, Hahn and Inoue (2001) «transforms the entire distribution of the portfolio returns into a single number, which investors have found useful and easily interpreted as a measure of market risk».

The object of VaR is to determine a distribution of the end-of-period portfolio taking into account the probable changes in the market risk factors, research in this matter is reported in Dowd (1998) and Duffie and Pan (1997).

Ahlgrim (1999) defined Value-at-Risk (VaR) as a probabilistic measure of the losses that are expected over a period of time under normal market conditions. Given a confidence level defined by a probability, losses over the defined horizon will exceed the VaR only a small percentage of the time. VaR is essentially a α-percentage quantile of the conditional distribution of the portfolio returns.

In actuarial terms, Wirch (1999) defines VaR, or VaR capital requirement, as quantile reserve, often using the 5% percentile of the loss distribution, using the empirical loss distribution over some appropriate time period.

In spite of its definition, the main goal of VaR is to quantify the uncertain amount which may be lost on a portfolio over a given period of time with a certain confidence level.

There are several models for calculating VaR. The existing models differ in the methodology they use, the assumptions they make and the way they are implemented. VaR models can be particularly different in the way they address the problem of the portfolio estimation, leading to the essential question: how to forecast the quantiles?

The different approaches that are used to model their variability distinguish VaR methodologies (see Manganelli and Engle, 2001);

i) Non Parametric (Historical Simulation);
ii) Parametric (Risk Metrics and GARCH);
iii) Semiparametric (Extreme Value Theory – EVT)

During several years, Risk Managers preferred choice was the use of non parametric techniques, mainly the Historical Simulation (HS) for estimating VaR (see Giannopoulos, 2002). Based on the return of the portfolio value, VaR is the percentile that corresponds to the VaR probability. The changes in the risk portfolio are associated only with the historical experience of the portfolio. For this non-parametric methodology, the final quantiles are under the assumption that any return in a particular period is equally likely. This method is relatively simple to implement since it does not make any distributional assumption about portfolio returns (see Danielsson and Vries, 1997; Dowd, 1998; Manganelli and Engle, 2001). It does not specify any assumptions about valuation models or the stochastic structure of the market.

The main problem with this technique is that VaR predictions consider all historical data equally relevant. As mention by Manganelli and Engle, (2001), «the distribution of portfolios returns does not, therefore, change within the window». Giannopoulos (2002) referred «that leaving out the highly volatile market conditions that may have occurred a little earlier than the beginning of the data window will make a huge difference in VaR prediction».

Historical simulation is based on an independent and identically distributed (i.i.d.) assumption, which is known to be incorrect under financial data; this limitation is seen in Sarma (2003).

Regarding the parametric models, Financial Institutions are using ARCH (autoregressive conditional heteroscedasticity) models and the related GARCH (generalised autoregressive conditional heteroscedasticity) formulations because they capture volatility persistence in a simply way (see Duffie and Pan, 1997). Models such as RiskMetrics and GARCH offer a specific parameterisation for the behaviour of returns (see Manganelli and Engle, 2001).

The ARCH models were introduced by Engle (1982) and express the conditional variance as a linear function of the past squared innovations (see Angelidis, Benos and Degiannakis , 2003).

A high order for the ARCH process was needed in order to grab the dynamic of the conditional variance. Reducing from infinite estimated parameters to two, the Generalized

ARCH (GARCH) developed by Bollerslev (1986) was the answer to the infinite parameters.

In GARCH models the conditional variance depends not only on the latest innovations, but also on the previous conditional variance. The simplest GARCH model is given by

$$u_t = \sigma_t \varepsilon_t \qquad \varepsilon_t \sim i.i.d.(0,1)$$
$$\sigma_t^{\ 2} = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2$$

where $\sigma_{t-1}^{\ 2}$ is the conditional variance using information up to time *t-1* and u_{t-1} the latest innovations. To achieve the conditional variance positive the following conditions must be verified, $\omega > 0$ and $\alpha, \beta \geq 0$.

In these models the sum of parameters $\alpha + \beta$ measures the persistence. The persistence parameter indicates the rate at which the multiperiod volatility forecast reverts to its unconditional mean (see Campbell, Lo and Craig, 1997).

The particular specification of the variance equation and the assumption ε_t i.i.d., are the two essential elements of the model. The first one it is due to the characteristics of financial returns and the second one is a necessary mechanism to estimate the unknown parameters (see Manganelli and Engle, 2001). An additional step is the specification of the distribution of the u_t. The most used distribution is the standard normal (see Nicolau, 2007). After this distribution assumption being defined, becomes possible to write down a likelihood function and get an estimate of the unknown parameters (see Manganelli and Engle, 2001).

The flexibility of ARCH modelling produced the developing of several volatility models. Extensive researches have focused on evaluating other volatility measures that improved conditional volatility forecasts. Developed by Glosten, Jagannathan and Runkle (1993), the GJR-GARCH as well known Threshold GARCH (T-GARCH), is the most common model used among asymmetric volatility. It was proposed using a dummy variable for negative shocks in the GARCH model.

Shock returns volatility reacts differently to positive and negative movements but generally when the asset price rises up, the variance of the return gets down. Although, daily returns are uncorrelated while the squared returns are strongly autocorrelated, letting that periods of persistent high volatility are followed by periods of persistent low volatility. This is the so called asymmetric effects, one of the most important characteristic of the financial data. Black (1976) called this phenomenon as Leverage Effect.

The RiskMetrics technique is a particular case of the GARCH family; the volatility under this parametric technique uses a particular autoregressive moving average process: Exponentially Weighted Moving Average (EWMA), for the price model, representing the finite memory of the sample.

RiskMetrics follows the assumption that returns are conditionally[1] normally distributed. This approach is a special case of the GARCH (1,1) process, where $\omega = 0$ and $\alpha + \beta = 1$, ${\sigma_t}^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) u_{t-1}^2$.

[1] Conditionally means: conditional on the prices set at time t, which usually consists of the past returns prices at time t

RiskMetrics considers the parameter of the model: λ (0< λ<1), usually set equal to 0.94 for daily data or 0.97 for monthly data, and assumes that standardised residuals are normally distributed, see J.P.Morgan (1996).

Under RiskMetrics methodology, VaR measure has only a few unknown parameters, which are simply calibrated and work quite well in common situations. However, several studies such as Christoffersen (1998) and Danielsson and Vries (1997) have found significant improvements when some modifications from the rigid RiskMetrics were explored.

Among the literature, the main findings related to both GARCH and RiskMetrics refer that both underestimate the VaR. This is due to the assumption of normality of the standardised residuals that are not quite consistent with the general behaviour of financial returns. There are some study's were these methods allow a complete identification of the distribution of returns and were an improvement of their performance has been achieved putting normal distribution assumption aside, see McNeil, and Frey (2000).

An alternative measure of market risk has been proposed, namely Extreme Value Theory (EVT). According to Sousa (1999), EVT is the appropriate methodology for tails with significant levels equal or less than 1%. According to his study, the semi parametric technique EVT is the best practice to model stress situations and also atypical situations.

As shown, for instance, in Danielsson and Vries (1997), models based on conditional normality are not well-matched to estimating large quantiles. The estimation of return distribution of financial time series through Extreme Value Theory (EVT) is a topical issue that has originated several works; see Embrechts (1998), Lauridsen (2000), Manganelli and Engle (2001).

The purpose of this dissertation was not to test the use of small confidence levels, therefore the EVT via was not used; however literature references are mentioned for this matter.

The most common time horizons used by commercial and investment banks to achieve VaR's are one day, one week, and two weeks. The Basle Committee and Banking Supervision (2001), mandate that banks using VaR models should control market risk using a holding period of two weeks and a confidence level of 1%. On the opposite, Institutional Investors holding long periods VaR figures are performed from one month to several years. The time horizon or the holding period can vary a lot in different applications; see Christoffersen (2001) and Jorion (1997).

If VaR is used to establish capital requirements, then regulators must decide the appropriate time of period ahead. As asset management firms, given the conservative view of accounting and its focus on liquidation values, it is appropriate to have a capital standard based on a short time period, such as one month or less. For example, due to the long-term of life insurance contracts it raises a potential difficulty in determining the appropriate time for a VaR calculation. The use of daily data to estimate volatilities among assets may not be valid over long time horizons. Giannopoulos (2002) refers the main issues to consider when using VaR with long horizons.

Backtests first performed by J.P.Morgan (1996) determine that Risk Managers should back test all models. Illustrated by the Basel rules, the Backtesting technique will evaluate how a model actually performed for a given period versus what was predicted, representing how often the actual losses have exceeded the level predicted by VaR. Backtesting will verify the accuracy, certifying that models are not systematically biased.

Even being one of the best practices in financial market, Value-at-Risk has some established criticisms. VaR only gives the upper value of the losses that can occur with a given frequency and, VaR does not reflect the potential size of the loss given that a loss exceeding the upper bound has occurred. Artzner (1999) shows that VaR as a measure of market risk has various theoretical deficiencies; Artzner refers «this situation in general occurs in portfolios containing non linear derivatives». Other critical statements related to VaR are to be found in Danielsson and de Vries (1997).

3. Technical Background and Data analysis

The aim of this dissertation is to evaluate the three methodologies (one non parametric and two parametric techniques) to measure Value-at-Risk, evaluating if they are good VaR estimators. The next chapters tend to resume the technical background and the data analysis used to achieve the final goal.

3.1. Estimating Volatility

Let P_t be the price of a portfolio at a time *t*. The observed return at time *t* is given by $r_t = \ln(P_t / P_{t-1})$, denoting the continuously compound rate of the return from time *t-1* to *t*. For a holding period *h*, the aggregate return at time t can be written by

$$R_{t,h} = \ln(P_{t+h-1} / P_{t-1}) = r_t + \ldots.. + r_{t+h-1}.$$

The historical information produced by the process $\{P_t\}$, namely the $\Im_t$, is the σ-algebra generated by $P_t, P_{t-1}, \ldots.$ The value of the portfolio at time *t+h* will be $P_{t+h} = P_t \exp(R_{t+1,h})$.

According to Value-at-Risk definition, the potential loss of a portfolio over a predetermined holding period *h* with a predefined confidence level (1-α), see Dowd (1998), is mathematical defined by,

$$prob(R_{t+1,h} > q_{t+1,h} \mid \Im_t) = 1 - \alpha$$

$$\Leftrightarrow$$

$prob(R_{t+1,h} < q_{t+1,h} \mid \Im_t) = \alpha$, were, $q_{t+1,h}$ is the α-quantile of the conditional distribution of $R_{t+1,h}$.

VaR, with a probability of α, is given by $VaR = P_t \tilde{q}_{t+1,h}$ (see Dowd, 1998 and Jorion, 1997). At time *t*, P_t is known, the unknown figure is the quantile of the conditional distribution. Several methods could be considered; in this study three different methodologies were used.

In practice, the selection of $h \neq 1$ period makes some complications in the estimation of VaR, see Wong and So (2001). Being $F_{n,t}(.)$ the cumulative distribution function for *h*-period return $R_{n,h}$ given $\Im_t$, i.e., $F_{n,t}$ (x) = Pr($R_{n,h} \leq x \mid \Im_t$).

To achieve VaR, the inverse of $F_{n,t}$ (.) for a certain confidence level is needed. As referred by Nicolau (2007) and Wong and So (2001), $F_{n,t}$ (.) is generally intractable, especially when *h* is large. The conditional distribution of $R_{n,h}$ given $\Im_t$ is written as

$$F_{n,h}(x) = \int_{R_{n,h \leq x}} f(R_{n,h} \mid \Im_{n+h-1}) \prod_{i=1}^{h-1} f(r_{n+i} \mid \Im_{n+i-1})\, d(r_{n+1},....,r_{n+h}) \ .$$

The evaluation of the inverse of the error distribution has to involve high-dimension integration. Generally the exact value is usually unavailable when h is greater then one.

As referred by Nicolau (2007) and Wong and So (2001) that will be adopt in this dissertation,

$$R_{n,h} \mid \Im_n \sim N\left(E\left[R_{n,h} \mid \Im_n\right], \mathrm{var}\left[R_{n,h} \mid \Im_n\right]\right)$$

3.1.1. Historical Simulation

According to this non-parametric method, the changes in the risk portfolio are related with the historical past experience of the portfolio. The final quantiles imply the assumption that any return in a particular period is equally likely. This method is relatively simple since it doesn't make any distributional assumption about portfolio returns. The past returns are used to predict future returns, see Danielsson and Vries (1997). Some details under the historical volatilities can be seen also in Duffie and Pan (1997).

3.1.2. RiskMetrics

The estimation of volatility using the RiskMetrics technique for one period return is defined by $\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2$ with $\lambda = 0.94$. By iterating it comes, $\sigma_t^2 = (1-\lambda)\left\{r_{t-1}^2 + \lambda r_{t-2}^2 + \lambda^2 r_{t-3}^2 + ...\right\}$ reflecting the exponential smoothing. For multiple periods return, the square root is frequently used $\sigma_{t+h|t} = \sqrt{h}\sigma_{t+1|t}$.

3.1.3. AR(1)+TGARCH

It was assumed that time series r_t are decomposed into two different parts; the predictable (conditional mean) and the unpredictable component, which is

$r_t = E[r_t \mid \Im_{t-1}] + u_t$, where $\Im_{t-1}$ is the available information at time *t-1*, $\mu_t = E[r_t \mid \Im_{t-1}]$ is the conditional mean and u_t is the unpredictable part, or also known as the innovation process.

The predictable component, conditional mean return was considered as a k-th order autoregressive process, AR(k), defined by

$$AR(k) \Rightarrow \mu_t = E[r_t \mid \Im_{t-1}] = c + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \ldots + \phi_k r_{t-k} .$$

The autocorrelation (AC) and partial autocorrelation (PAC) functions were analysed, using the Eviews[2], to achieve the specify order lag, characterizing the pattern of the temporal dependence in series. By definition the PAC of a autoregressive process of order k, AR(k), cuts off at lag k.

From Figure1 below, the output from the Eviews, provides some evidence that time series considered is a 1-th order autoregressive process, AR(1) or a 1-th order moving average process, MA(1). For lags higher then one, the autocorrelation is within the bounds, which means it is not significantly different from zero at a 5% confidence level. Due to the final results AR(1) provides the best out-of-sample forecast for the studied portfolio. Therefore the predictable component, was considered as a 1-th order autoregressive process, AR(1).

[2] Eviews – Econometric Software version 5

Figure 1 – Auto and Partial correlations - Eviews output

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Correlogram of Standardized Residuals

Sample: 1 610
Included observations: 610

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.118	0.118	8.4864	0.004
		2	-0.020	-0.034	8.7326	0.013
		3	0.002	0.009	8.7351	0.033
		4	-0.009	-0.011	8.7849	0.067
		5	0.023	0.026	9.1170	0.104
		6	0.030	0.024	9.6699	0.139
		7	-0.014	-0.020	9.7946	0.201
		8	-0.061	-0.056	12.070	0.148
		9	0.006	0.019	12.090	0.208
		10	-0.044	-0.050	13.269	0.209
		11	-0.034	-0.023	13.986	0.234
		12	0.003	0.006	13.993	0.301
		13	-0.002	-0.001	13.996	0.374
		14	-0.003	-0.001	14.002	0.450
		15	0.065	0.066	16.646	0.340
		16	-0.003	-0.019	16.651	0.409
		17	0.016	0.025	16.807	0.468
		18	0.042	0.031	17.946	0.459
		19	0.033	0.026	18.649	0.480
		20	0.040	0.031	19.639	0.481

The unpredictable component was expressed as an ARCH model, $\sigma^2{}_t$ is the conditional variance, being positive, changing with time and is a measurable function at time *t-1*, see Angelidis and Benos and Degiannakis (2003).

Thus, the final model used was specified as:

$$r_t = c + \phi r_{t-1} + u_t$$

$$u_t \mid \Im_{t-1} \sim N(0, \sigma_t^2)$$

$$u_t = \sigma_t \varepsilon_t .$$

In order to capture the asymmetry exhibited of returns, reflecting the effect of good and bad news on volatility, the so called GJR-GARCH model, developed by Glosten, and Jagannathan and Runkle (1993), was developed, and is defined by,

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \gamma u_{t-1}^2 \mathrm{I}_{\{u_{t-1<0}\}} + \beta \sigma_{t-1}^2 \text{ , where}$$

$$\mathrm{I}_{\{u_{t-1<0}\}} = \begin{cases} 1 & if \quad u_{t-1} < 0 \\ 0 & if \quad u_{t-1} \geq 0 \end{cases} .$$

For the time series considered in this study, the positive model variations, which are good news, are not statistically significant for a 5% confidence level, see below table 1, output of Eviews. The positive variations do not influence the volatility under this portfolio.

Table 1 - Statistics TGARCH Model EVIEWS Output

Dependent Variable: R_100
Method: ML - ARCH (Marquardt) - Student's t distribution
Sample (adjusted): 2 610

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.015843	0.004687	3.380194	0.0007
R_100(-1)	0.096208	0.037940	2.535765	0.0112
	Variance Equation			
C	0.002936	0.001164	2.523394	0.0116
RESID(-1)^2	-0.046590	0.034677	-1.343529	0.1791
RESID(-1)^2*(RESID(-1)<0)	0.202470	0.076014	2.663593	0.0077
GARCH(-1)	0.751229	0.091318	8.226554	0.0000
T-DIST. DOF	6.556926	1.961781	3.342333	0.0008

R-squared	0.013579	Mean dependent var	0.014534
Adjusted R-squared	0.003747	S.D. dependent var	0.126000
S.E. of regression	0.125763	Akaike info criterion	-1.400086
Sum squared resid	9.521476	Schwarz criterion	-1.349376
Log likelihood	433.3263	Durbin-Watson stat	1.929422

Therefore, the final model used in this study was a particular GJR-GARCH model,

$$\sigma_t^2 = \omega + \gamma u_{t-1}^2 \mathrm{I}_{\{u_{t-1}<0\}} + \beta\sigma_{t-1}^2 ,$$

where

$$\mathrm{I}_{\{u_{t-1}<0\}} = \begin{cases} 1 & if \quad u_{t-1} < 0 \\ 0 & if \quad u_{t-1} \geq 0 \end{cases}.$$

3.2. Estimating Value-at-Risk

3.2.1. Historical Simulation

The Historical Simulation (HS) approach simplifies the procedure to obtain Value-at-Risk. No distribution assumption is needed and, as previously mentioned, the past returns are used to predict future returns. VaR at a $\alpha\%$ confidence level will be the $\alpha\%$ quantile of the worst outcomes. If the corresponded number falls between two consecutive returns, then an interpolation rule is applied (see Manganelli and Engle, 2001).

Mathematically, VaR for one holding period is defined by, see Nicolau (2007),

$$prob(P_{n+1} - P_n < -VaR_{n,n+1,\alpha} \mid \Im_n) = \alpha \Leftrightarrow prob(R_{n+1}P_n < -VaR_{n,n+1,\alpha} \mid \Im_n) = \alpha \Leftrightarrow$$

$$\Leftrightarrow prob(R_{n+1} < \frac{-VaR_{n,n+1,\alpha}}{P_n} \mid \Im_n) = \alpha \Leftrightarrow prob(R_{n+1} < q_\alpha \mid \Im_n) = \alpha ,$$

assuming,

$$prob(r_{n+1} < q_\alpha \mid \Im_n) = prob(r_{n+1} < q_\alpha),$$

Value-at-Risk can be estimated, $V\hat{a}R_{n,n+1,\alpha} = -\tilde{q}_\alpha P_n$ where the P_n represents the portfolio amount at time *t*, and $\tilde{q}_\alpha$ represents the empirical α-quantile of the returns $\{R_{n+h}(h), n = 1,2,...\}$.

3.2.2. RiskMetrics

To achieve VaR through the RiskMetrics methodology, according to J.P.Morgan (1996), two different steps must be carried out. The first one requires the need to estimate the volatility for holding portfolio for one day before converting it into the volatility for multiple days. The second one requires the compute of the quantile of the standardised return processes, based on the assumption that the process follows a standard normal distribution.

For one holding period, the daily VaR, for the confidence level α, 5% and 1%, is calculated by multiplying the volatility estimated on a given day, with the (1-α) quantile of the standard normal distribution, according to RiskMetrics techniques.

To achieve a multiperiod forecast it applies the use of the “squared-root-of-time”; $\sigma_{t+h|t} = \sqrt{h}\sigma_{t+1|t}$ derived on the assumption of uncorrelated returns, see J.P.Morgan (1996).

For *h* periods the Value-at-Risk is given by $VaR_{n,n+h,\alpha} = -P_n z_\alpha \sqrt{h}\sigma_n$.

3.2.3. AR(1) + TGARCH

The GJR-GARCH model considered in this study is given by,

$$r_t = c + \phi r_{t-1} + u_t \qquad \text{, with}$$

$$u_t = \sigma_t \varepsilon_t \qquad\qquad \sigma_t^2 \to TGARCH$$

$$\sigma_t^2 = \omega + \gamma u_{t-1}^2 \mathrm{I}_{\{u_{t-1}<0\}} + \beta \sigma_{t-1}^2 \text{, (volatility)}$$

where

$$\mathrm{I}_{\{u_{t-1}<0\}} = \begin{cases} 1 & if \quad u_{t-1} < 0 \\ 0 & if \quad u_{t-1} \geq 0 \end{cases}.$$

Under the ARCH models the maximum likelihood estimation is frequently used. Following the assumption, for $(r_t - \mu_t)$, of independently and identically distributed standardized innovations (i.i.d.), and being *f* the density function, the log-likelihood function based on standardised t-student distributed innovations is given by

$$L(\theta) = \sum_{t=1}^{n} L_t(\theta) \text{ , where}$$

$$L_t(\theta) = \log f(r_t \mid \Im_{t-1}) = \log\left(\frac{1}{\sqrt{\sigma_t^2 \pi(\nu-2)}}\right) \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left\{1 + \frac{\left(\frac{y_t - r_t}{\sigma_t}\right)^2}{\nu-2}\right\}^{-\frac{\nu+1}{2}}$$

$$= -\frac{1}{2}\log\sigma_t^2 - \frac{1}{2}\log\pi - \frac{1}{2}\log(\nu-2)$$

$$+ \log\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} - \frac{\nu+1}{2}\log\left(1 + \frac{1}{\nu-2}\frac{(r_t-\mu_t)^2}{\sigma_t^2}\right)$$

$\Gamma(.)$ is the Gamma function and ν is the number of degree of freedom, and μ_t, σ_t^2 are the conditional mean, and variance of the model respectively. Under this model the unknown parameters are denoted by θ. To achieve the real parameter vector, the maximum likelihood estimator $\hat{\theta}$, is obtained by maximizing the equation above.

The unknown parameters for the presented model are $\theta = (c, \phi_1, \omega, \gamma, \beta, \nu)'$ under the usual restrictions of the GARCH models, and with $\gamma > 0$.

The distribution of the quantile for *h=1* (one period forecasting) is known (z_α), under the studied model, is a t-Student (standardised), but for periods higher than one the distribution is generally unknown. Regarding literature, for *h>1,* the quantile of the normal distribution was taking into account.

Following the assumption that the conditional portfolio returns for *h>1* holding periods has a normal distribution (see Nicolau, 2007);

$$r_{n+h}(h) \mid \Im_n \approx N\left(E\left[r_{n+h}(h) \mid \Im_n\right], \sqrt{\operatorname{var}\left[r_{n+h}(h) \mid \Im_n\right]}\right)$$

leads to

$$VaR_{n,n+h,\alpha} = -(E\left[r_{n+h}(h) \mid \Im_n\right] + z_\alpha \sqrt{\operatorname{var}\left[r_{n+h}(h) \mid \Im_n\right]})P_n \, .$$

It is known that

$$E\left[r_{n+h}(h) \mid \Im_n\right] = \mu_{n+1,n} + \ldots + \mu_{n+h,n} \, ,$$

working with a AR(1) structure

$$\mu_{n+h,n} = c\frac{1-\phi^h}{1-\phi} + \phi^h y_n$$

therefore,

$$E\left[r_{n+h}(h)\,|\,\Im_n\right]=c\frac{1-\phi^1}{1-\phi}+\phi^1 y_n+\ldots+c\frac{1-\phi^h}{1-\phi}+\phi^h y_n$$

using the Mathematica Software, the following is obtain;

$$E\left[r_{n+h}(h)\,|\,\Im_n\right]=\frac{y_n(-1+\phi)\phi(-1+\phi^h)+c(h(1-\phi)+\phi(-1+\phi^h))}{(-1+\phi)^2}\,.$$

It was also necessary to calculate the $\mathrm{var}\left[r_{n+h}(h)\,|\,\Im_n\right]$. For the specific model in this study

$$\sigma_n^2=\omega+\gamma u_{n-1}^2\mathrm{I}_{\{u_{n-1<0}\}}+\beta\sigma_{n-1}^2$$

for one period forecasting, it comes

$$\sigma_{n+1,n}^2=\omega+\gamma u_n^2\mathrm{I}_{\{u_{n<0}\}}+\beta\sigma_n^2$$

for two periods forecasting,

$$\sigma_{n+2,n}^2=\omega+\left(\frac{\gamma}{2}+\beta\right)\sigma_{n+1,n}^2$$

for *h forecasting periods,*

$$\sigma_{n+h,n}^2=\omega+\delta\sigma_{n+h-1,n}^2\text{ , where }\delta=\frac{\gamma}{2}+\beta$$

iterating $\sigma_{n+h,n}^2$ in order of $\sigma_{n+1,n}^2$, it appears

$$\sigma_{n+h,n}^2=\frac{\omega}{1+\delta}+\delta^{h-1}(\sigma_{n+1,n}^2-\frac{\omega}{1+\delta})$$

taking to the final expression,

$$\mathrm{var}\left[r_{n+h}(h)\,|\,\Im_n\right]=\sum_{k=1}^{h}\left(\left(\sum_{j=0}^{h-k}\phi^{j}\right)^{2}\left(\frac{\omega}{1-\delta}+\delta^{h-1}\left(\sigma^{2}_{n+1,n}-\frac{\omega}{1+\delta}\right)\right)\right)$$

through the use of the Mathematica Software, $\mathrm{var}\left[r_{n+h}(h)\,|\,\Im_n\right]$ reduces to

$$-\frac{1}{(-1+\phi)^{2}(-1+\delta)}\Bigg(\ h\omega+\frac{\phi(-1+\phi^{h})(-2-\phi+\phi^{1+h})}{-1+\phi^{2}}+\sigma^{2}_{n+1,n}(1-\delta^{h})+\frac{\omega(1-\delta^{h})}{-1+\delta}+$$

$$+(\omega+\sigma^{2}_{n+1,n}(\delta-1))x\left(\frac{\phi^{2+2h}\left(\left(\frac{\delta}{\phi^{2}}\right)^{h}-1\right)}{\phi^{2}-\delta}-2\frac{\phi^{1+h}\left(\left(\frac{\delta}{\phi}\right)^{h}-1\right)}{\phi-\delta}\right)\Bigg)$$

(see Nicolau, 2007, section 11.3.3).

3.3. Backtesting

The main goal was to test and analyse the three different approaches for the forecasting techniques, in a risk management atmosphere, i.e, measuring risk. The quality of the volatility forecasts and the respectably independence on forecasts will affect the quality of the forecasted VaR. The purposes of Backtests methodology is to monitor VaR forecasts and after that evaluate volatility models, being sure that models are not systematically biased.

Backtesting tests will verify volatility forecasts as good VaR estimators testing the rule that the values exceeding VaR are independent and identically distributed (being a Bernoulli distribution with probability of success α).

Introduced by Christoffersen, Diebold and Schermann (1998), the likelihood ratio test was created for testing the independence, and also for testing the $prob(I_t = 1) = \alpha \Leftrightarrow E[I_t] = \alpha$, generally known by correct unconditional coverage, where I_t is the indicator event. The indicator event is defined as the returns that exceed VaR by

$$I_t = \begin{cases} 1 & if \ r_t < -VaR_{t,t-1,\alpha} \\ 0 & if \ r_t \geq -VaR_{t,t-1,\alpha} \end{cases}.$$

For testing the independence, one of the possible ways can be through the known *runs test*, the hypothesis testing is H_0: $\{I_t\}$ is independent and identically distributed (i.i.d), see Nicolau (2007). For samplings with $n_0 > 20$ or $n_1 > 20$ the statistic test is given by $prob(|Z| > |z_{obs}|)$, where $Z = \frac{X - E[X]}{\sqrt{\mathrm{var}[X]}} \xrightarrow{d} N(0,1)$, with

$$E[X] = \frac{2n_0 n_1}{n} + 1 \quad \text{and} \quad \mathrm{var}[X] = \frac{2n_0 n_1 (2n_0 n_1 - n)}{n^2 (n-1)}.$$

The limits under X are; $\max X = \begin{cases} 2\min\{n_0, n_1\} & if \ n_0 = n_1 \\ 2\min\{n_0, n_1\} + 1 & if \ n_0 \neq n_1 \end{cases}.$

The sampling figures are represented by $n = n_0 + n_1$, with n_0, n_1 number of zeros and ones, respectively.

The unconditional coverage is tested through the hypothesis $H_0 : E[I_t] = \alpha$, where the maximum likelihood ratio is defined by

$$LR = -2\log\frac{\alpha^{n_1}(1-\alpha)^{n_0}}{\hat{\alpha}^{n_1}(1-\hat{\alpha})^{n_0}},$$ with $\hat{\alpha} = \frac{n_1}{n}$, α is the defined confidence level, 5% or 1%.

Under the null hypothesis, the statistic test LR is approximately distributed to χ_1^2, chi-squared distribution with one degree of freedom (see Nicolau, 2007). For analysing the statistics tests it is equivalent to analyse the *p-value*.

3.4. Data analysis

A real Portuguese Life Insurance portfolio was used. This portfolio is linked to several financial assets; stocks, stock indices, foreign currency, etc. For simplicity, in this dissertation it was assumed as a unique portfolio risk.

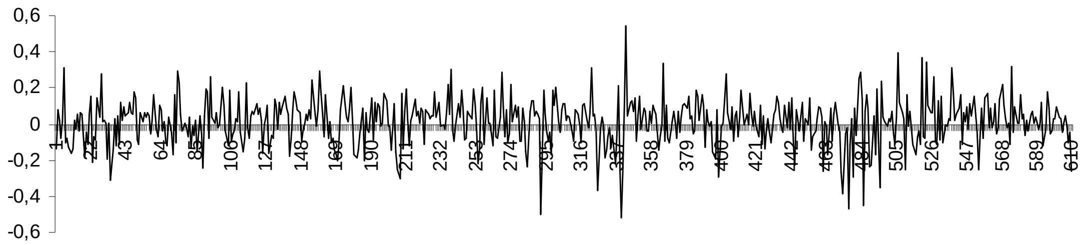
The daily figures were obtained from 22nd of June 2004 until 28th of November 2006, before this data there were no daily figures available on the system of the company, only monthly figures. Excluding weekends and holidays in Portuguese market were used 610 returns. Following Nicolau, (2007), it was considered continuous returns. The final returns were considered times 100 (*R_100*), due to the use of the software Gauss routine. Most of the contracts will end during this current year, and the totally at the end of the next year. For the asset management firm that manages funds, the control of the market risk is empirical. Therefore this dissertation aims to study if the volatility measures can be used to predict a good VaR estimator.

Three different methodologies were used to achieve Value-at-Risk for two different confidence levels, 5% and 1%, and forecasted for three holding periods, one day, one and two weeks. The final conclusion about volatility measure was achieved through the Backtesting technique.

As defined on the technical background, the P_n represents the portfolio amount at time t. For simplification in this study the portfolio amount was considered equal to one.

The total 610 returns are presented on figure 2 below,

Figure 2 - Portfolio daily Returns



The volatility clusters (Leverage effect), strong (weak) variations are more probable to be followed by strong (weak) variations, are generally associated to financial data, the evidence on the portfolio returns studied is noted.

The table 2 below shows a negative skewness, meaning the distribution is asymmetric, having heavier tails. The kurtosis exceeds 3, generally the kurtosis of the normal distribution, meaning that distribution is leptokurtic relative to the normal (see Newbold, 2003).

Table 2 - Return Statistics for the entire sample

	Observations	Mean	Variance	Maximum	Minimum	Kurtosis	Skewness	Jarque-Bera	Prob.
R_100	610	0,01429	0,01589	0,54428	-0,52027	4,9222	-0,3477	1,0621	0,000

Table 2 summarises the main statistics and also the Jarque-Bera Statistic used for testing normality. The null hypothesis of normality is rejected at any level of confidence, combined with the evidence of the kurtosis higher than three and the negative skewness.

Table 3 - Parameters Estimated for Normal distribuiton and for t-Student distribuiton

Distribution	c	ϕ_1	ω	β	γ	v	Log Likelihood
t-Student	0.016833	0.09773	0.002701	0.739203	0.166594	6.556926	432.3661
prob	0.0003	0.0160	0.0293	0.0000	0.0209	0.0006	-
Normal	0.013947	0.100477	0.001973	0.805615	0.126706	-	420.6259
prob	0.0062	0.0155	0.0782	0.0000	0.0210	-	-

The estimated parameters, obtained by Eviews, for normal and t-student distribution are presented on table 3 above. The model is well specified using GJR-GARCH model combined with T-Student distribution all estimated coefficients are statistically significantly. For normal distribution one of the GARCH parameters is excluded ω.

4. Estimation Results

Considering the particular GJR-GARCH model and the t-Student distribution, the final model results are presented on table 4 below. The modelling estimation was achieved by Eviews,

Table 4 - TGARCH Model EVIEWS Output

Dependent Variable: R_100
Method: ML - ARCH (Marquardt) - Student's t distribution
Sample (adjusted): 2 610

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.016833	0.004623	3.641274	0.0003
R_100(-1)	0.09773	0.040566	2.409186	0.0160
	Variance Equation			
C	0.002701	0.001239	2.180091	0.0293
RESID(-1)^2*(RESID(-1)<0)	0.166594	0.072139	2.309341	0.0209
GARCH(-1)	0.739203	0.097380	7.590920	0.0000
T-DIST. DOF	6.355273	1.852825	3.430045	0.0006

R-squared	0.013240	Mean dependent var	0.014534
Adjusted R-squared	0.005058	S.D. dependent var	0.126000
S.E. of regression	0.125681	Akaike info criterion	-1.400217
Sum squared resid	9.524743	Schwarz criterion	-1.356751
Log likelihood	432.3661	Durbin-Watson stat	1.931512

The Backtesting methodology was applied analysing all history returns from the beginning of 2006. The Backtesting results for one holding period, for both confidence levels and for the three methodologies are presented below in table 5,

Table 5 - Backtesting results for one holdind period

α = 5%

Independence Test

Method	X	Medium	Var	n_o	n_1	Z_{obs}	p-value
HS	26	29,0263	3,3368	213	15	-1,6567	0,0976
RM	20	21,9386	1,8391	217	11	-1,4295	0,1529
TGARCH	17	18,2895	1,2407	219	9	-1,1577	0,2470

unconditional coverage

Method	α estimated	n_o	n_1	RV	p-value
HS	0,0658	213	15	1,0933	0,2957
RM	0,0482	217	11	0,0149	0,9027
TGARCH	0,0395	219	9	0,5715	0,4497

α = 1%

Independence Test

Method	X	Medium	Var	n_o	n_1	Z_{obs}	p-value
HS	7	6,9211	0,1284	225	3	0,2204	0,8256
RM	14	16,4386	0,9820	220	8	-2,4609	0,0139
TGARCH	3	2,9912	0,0087	227	1	0,0941	0,9251

unconditional coverage

Method	α estimated	n_o	n_1	RV	p-value
HS	0,0132	225	3	0,2089	0,6476
RM	0,0351	220	8	8,7904	0,0030
TGARCH	0,0044	227	1	0,9189	0,3378

For one holding period and for a 1% confidence level, the RiskMestrics does not verify the independence test.

Table 6 - Backtesting results for one week holdind period

α = 5%

Independence Test

Method	X	Medium	Var	n_o	n_1	Z_{obs}	p-value
HS	5	21,9196	1,8687	213	11	-12,3773	0
RM	10	23,7143	2,2118	212	12	-9,2215	0
TGARCH	12	25,4911	2,5799	211	13	-8,3993	0

unconditional coverage

Method	α estimated	n_o	n_1	RV	p-value
HS	0,0491	213	11	0,0038	0,9510
RM	0,0536	212	12	0,0588	0,8083
TGARCH	0,0580	211	13	0,2902	0,5901

α = 1%

Independence Test

Method	X	Medium	Var	n_o	n_1	Z_{obs}	p-value
HS	7	14,5625	0,764	217	7	-8,6519	0
RM	5	8,8571	0,2416	220	4	-7,8472	4E-15
TGARCH	7	16,4286	0,9983	216	8	-9,4368	0

unconditional coverage

Method	α estimated	n_o	n_1	RV	p-value
HS	0,0313	217	7	6,5350	0,0106
RM	0,0179	220	4	1,1326	0,2872
TGARCH	0,0357	216	8	8,9984	0,0027

As referred, under the literature, VaR is generally calculated for different periods of time. Therefore it was also studied if forecasted volatilities are good VaR measure for the next five and ten holding periods, i.e., one week and two weeks. The results are present on table 6 for one-week holding period above, and on table 7 below for two weeks holding.

Table 7 - Backtesting results for two weeks holdind period

α = 5%

Independence Test

Method	**X**	**Medium**	**Var**	**n_o**	**n_1**	**Z_{obs}**	**p-value**
HS	9	21,8950	1,9069	208	11	-9,3380	0
RM	5	16,4155	1,0194	211	8	-11,3065	0
TGARCH	13	27,2100	3,031	205	14	-8,1621	0

unconditional coverage

Method	**α estimated**	**n_o**	**n_1**	**RV**	**p-value**
HS	0,0502	208	11	0,0002	0,9876
RM	0,0365	211	8	0,9193	0,3377
TGARCH	0,0639	205	14	0,8250	0,3637

α = 1%

Independence Test

Method	**X**	**Medium**	**Var**	**n_o**	**n_1**	**Z_{obs}**	**p-value**
HS	3	14,5525	0,7804	212	7	-13,0780	0
RM	3	6,9178	0,1335	216	3	-10,7227	0
TGARCH	3	12,6712	0,5713	213	6	-12,7951	0

unconditional coverage

Method	**α estimated**	**n_o**	**n_1**	**RV**	**p-value**
HS	0,0320	212	7	0,0002	0,9876
RM	0,0137	216	3	0,27129	0,3377
TGARCH	0,0274	213	6	4,5416	0,0331

As the periods are getting higher the independence is being lost. For the three methodologies and for both confidence levels there is the evidence that forecasted volatilities are not good VaR measures. Forecasted volatilities do not have the property of, the sequence of the events exceeding VaR behaves like an i.i.d..

5. Main Findings

The main objective of this study was to evaluate and analyse forecasted volatilities as good VaR estimators for a Financial Institution portfolio.

Through the use of three different methodologies for two different confidence levels and for three time holding periods, backtests were performed in order to achieve the main conclusions.

The backtests reveal that volatilities are good VaR measures for one holding period under the parametric technique AR(1)+GJR-GARCH for both confidence levels, and as well for the non parametric technique – Historical Simulation. RiskMetrics, the other parametric technique, turned out to be a good VaR measure only for one holding period at a 5% confidence level. For the other two holding periods forecasted volatilities revealed not to be good VaR measures.

The final figures for Value-at-Risk, for one holding period and for 95% confidence level are presented on table 8 below.

Table 8 - Value-at-Risk for one day holding period, 5% confidence level

α=5%	P_n	Historical Simulation	RiskMetrics	TGARCH
VaR h=1	**1**	-0,1904	-0,1246	-0,2656

On table 9 below, the final figures for Value-at-Risk for one holding period for 1% confidence level are presented.

Table 9 - Value-at-Risk for one day holding period, 1% confidence level

α=1%	P_n	Historical Simulation	RiskMetrics	TGARCH
VaR h=1	**1**	-0,3492	-0,1762	-0,4233

The well-known shortcoming, under the literature, on the non-parametric technique Historical Simulations it is the fact does not grab the dynamic of the conditional variance. The changes in the risk portfolio are associated only with the historical experience of the portfolio, and the quantiles rest on the assumption that any return in a particular period is equally likely.

Under the limitations of RiskMetrics methodology, referring it underestimates VaR due to the normality assumption that generally is not consistent with the general behaviour of financial data, the GJR-GARCH methodology reveals to be the most accurate measure to estimate VaR.

For further developments it would be interesting to evaluate other confidences levels, less than 1%, and estimating VaR through the use of semiparametric techniques such as Extreme Value Theory referred on the literature.

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Appendix*

Programming Code – AR(1)+TGARCH, one, five and ten holding periods, indicator backtesting

```
library cml,util,pgraph,est_etd,testes;
cls;

y = SpreadsheetReadM("dados rc", "d3:d612",1 );
y=y*100;

n=rows(y);
@ k=290; @
k=382;
alfa=.05;
missing={.};
Value_at_Risk=missing*ones(rows(y),1);
Indicador=missing*ones(rows(y),1);
controle={};
h=1;                    \* h=5 ;  h=10        */

b0=0.11889583|0.014438678|0.0027099201|0.15353301|0.74367327|7;\* Vector Inicial MV */
for i (k,n-h,1);
    {b,f0,grad,cov,retcode}=tStudent_garch_Sandra(b0,y[1:i]);
    if retcode /= 0;
        controle =controle|i;
    endif;
    b0=b;
    m=b[1]*y[i]; /* media*/

    m=(y[i]*(-1 + b[1])*b[1]*(-1 + b[1]^h) +
      b[2]*(h - h*b[1] + b[1]*(-1 + b[1]^h)))/(-1 + b[1])^2;

    u=y[i]-m; /* erro*/
    v_1=b[3]+b[4]*(u < 0)*u^2+b[5]*var[rows(var)];   /* variancia a um passo*/
    v_h=-((h*b[3] + (b[1]*(-1 + b[1]^h)*(-2 - b[1] + b[1]^(1 + h))*b[3])/(-1 +
            b[1]^2) +
        v_1*(1 - (b[4]/2 + b[5])^h) + (b[3] -
            b[3]*(b[4]/2 + b[5])^h)/(-1 + b[4]/2 + b[5]) + (b[3] +
            v_1*(-1 + b[4]/2 +
                b[5]))*((b[
                    1]^(2 + 2*h)*(-1 + ((b[4]/2 + b[5])/b[1]^2)^h))/(b[
                    1]^2 - b[4]/2 - b[5]) - (2*
                b[1]^(1 + h)*(-1 + ((b[4]/2 + b[5])/b[1])^h))/(b[1] -
                b[4]/2 - b[5])))/((-1 + b[1])^2*(-1 + b[4]/2 + b[5]))); /* variancia a h passos*/

    quantil=cdfni(alfa); /* quantil normal */
    Value_at_Risk[i+h]=-(m+quantil*sqrt(v_h));
    if sumc(y[i+1:i+h]) < -Value_at_Risk[i+h];
        indicador[i+h]=1;
        else;
        indicador[i+h]=0;
    endif;
endfor;

print "controle";; controle;
i=packr(indicador);
```

* The programming code was build with the collaboration of Professor João Nicolau

```
print "Media de I";;meanc(i);
xy(0,y~-Value_at_Risk);

call runs_test(I);
call teste_media_binomial(I,alfa);


cls;
value_at_risk~indicador;

proc (5)= tStudent_garch_Sandra(b0,y);
            local b,f0,grad,cov,retcode,media,var,z;


            /*
            _cml_Algorithm=4;*/
   __output=0;
            _cml_CovPar=3;


   __title= "MÉTODO DA MV  - Distribuição t-Student";

            z=packr(y~desfas2(y,1)~ones(rows(y),1));
   _cml_Bounds={0 1,-1e10 1e10,0 1e10,0 1e10,0 .99,3 1e10};

   _cml_ParNames = "fhi"|"beta"|"k"|"gama"|"delta"|"v";
            {b,f0,grad,cov,retcode } = cml(z,0,&logl_tStudent_garch_Sandra,b0);

            call cmlprt(b,f0,grad,cov,retcode);


            retp(b,f0,grad,cov,retcode);
endp;

/* Função de Verosimilhança */

proc logl_tStudent_garch_Sandra( b, z );
            local fhi,beta,k,gama,delta,u,u_des,v;

            fhi=b[1];
            beta=b[2];
            k=b[3];
   gama=b[4];
   delta=b[5];
   v=b[6];

            media=z[.,2:2] * fhi+z[.,3:3] * beta;
            u = (z[.,1] - media);
   u_des=0|u[1:rows(z)-1];

   var = recserar(k+gama*u_des^2 .*(u_des.<0),meanc(u[1:20]^2),delta);


            retp(-1/2*ln(var)-1/2*ln(pi)-1/2*ln(v-2)+ln(gamma((v+1)/2)/gamma(v/2))-(v+1)/2*ln(1+u^2 ./(var*(v-2)))
);
endp;
```